Thought Experiments as logical transformations In Transreal Logical Space: Re-Examining the Einstein, Podolsky and Rosen Paradox

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Submitted: 15 April 2019
Revised: 21 May 2019

Abstract

In this article, from the concepts of formal causality and logical transformation, defined with transreal numbers, I intend to re-analyze the famous Einstein, Podolsky and Rosen paradox (the EPR paradox), according to which Quantum Mechanics is incomplete. In order to make such an analysis of the paradox, I present a general definition of thought experiments, in terms of the concept of logical transformation in a transreal logical space, and show that the EPR paradox, in broad outlines, bases the incomplete character of Quantum Mechanics on the fact of not having a formal causality between the ideal and concrete worlds of quantum theory - these concepts, the"ideal and concrete worlds", by their turn, are inspired by the work of the American physicist Wolfgang Smith.

1 What is a Thought Experiment? A Very General Mathematical Approach

What is a thought experiment? This research tries to answer that question within the domain of transreal numbers, and by appealing to some metaphysical considerations that lead us to the concept of formal causality, an Aristotelian-Platonic notion.

This research is not a historical one, but intends to be a theoretical attempt to answer the question concerning what is the nature of a thought experiment.

First of all, I am considering that, at first glance, a thought experiment is a logical transformation (to be formally defined latter) that implies some theoretical conclusions from some ideal descriptions of the world to the real world. In every thought experiment, there is a nucleus that can be described as follows: an experiment in some ideal world W_i is mentally made by some researcher, and this experiment generates a conclusion about the real world W_r. In a very general way, a thought experiment could be seen as a relation E between two worlds, an ideal world and the real world. We can also stress that a thought experiment can be considered as a transformation T between an ideal world W_i and the real world W_r. In other words:

$$T(W_i) = W_r \iff E(W_i, W_r).$$

In the logical expression, above, we are dealing with the concept of world in a mathematical sense: a world is a *point* or a *vector* in some kind of logical space. Later, I will present some mathematical instruments, based on transreal numbers, that are sufficient to give a formal definition of relations between worlds.

In philosophical terms, a thought experiment is a relation or a transformation between worlds that are *similar* to each other. But such similarity lies in the fact that they share the same causal relations regarding some aspects of the real world: if the ideal world is organized by some causal relation C_i between their ideal components, then the causal relation C_r that organizes the real correspondents of the ideal components is similar to C_i.

Thus, in some sense, the causal similarity between ideal and real worlds is a postulate of the efficacy of the thought experiment; and I admit that this similarity is based upon the notion of formal causality between ideal and real world.

2 Formal Causality

For giving an idea of what shall be considered as formal causality between worlds, I will follow some ideas developed by the American Physicist Wolfgang Smith in his book *The Quantum Enigma. Finding the Hidden Key* [7]. In this work, Smith considers that there are two disconnected worlds in the physical realm, the *physical world*, a mathematically structured world of which mathematical-physical theories are testimonies, and the *concrete world*, the world in which apparatus of measurements and observers – people who make measurements – are located. The physical realm is the interaction of these two worlds, and physical theories guide us in how to deal with the concrete world.

My idea is that the disconnectedness between such worlds could be expressed mathematically by means of the transreal number nullity, Φ, the number that "measures" the indeterminate or the disconnectedness between two points: if the distance between two points is nullity, *then no information can physically travel between them.*

According to what is expressed above, the connection between *physical* and *concrete* worlds is not of a physical nature; but we can postulate some metaphysical relation between them, and such metaphysical connection I will call *formal causality*: every *true mathematical expression in the physical world "causes" a true expression in the concrete world.* In others words, a true mathematical theory in the realm of the physical world gives to us a true description of the concrete world [4].

3 The Concepts of Transreal Logical Space and Logical Transformation

Transreal numbers were created, around 1997, by James A. D. W. Anderson, an English Computer Scientist. For a modern treatment see [6]. The transreal numbers augment the real numbers with three new numbers, namely: negative infinity, $-\infty = -1/0$; positive infinity, $\infty = 1/0$; and nullity, $\Phi = 0/0$. The transreal numbers allow division by zero, a division forbidden in the realm of real numbers.

The transreal number, nullity, is the uniquely unordered, transreal, number with the property, $t \nless \Phi$ and $t \ngtr \Phi$ for all transreal numbers, t.

Before introducing an interpretation of a thought experiment within transreal logical space, it is necessary to present the concept of "total semantics", see [1], a semantics in which all logical possibilities are modelled with transreal numbers. Basically, the idea is the following:

a) If a proposition α has some degree of Truehood, then the transreal numbers r related to α is greater than zero: $r > 0$;

b) If a proposition α has some degree of Falsehood, then the transreal numbers r related to α is lower than zero: $r < 0$;

c) If a proposition α is Absolutely True, then the transreal number r related to α is ∞: $r = \infty$;

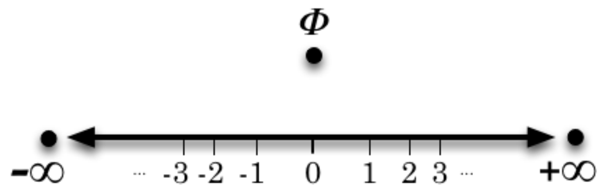
d) If a proposition α is Absolutely False, then the transreal number r related to α is $-\infty$: $r = -\infty$;

e) If a proposition α is neither True nor False (a gap), then the transreal number r related to α is Φ: $r = \Phi$;

f) If a proposition α is both True and False (a glut), then the transreal number r related to α is 0: $r = 0$.

If the propositions α and β have associated to them respectively the transreal numbers a and b, in such way that $a < b$, then we say β is truer than α; if $a > b$, then we say that β is falser than α.

Transreal numbers are shown in the following picture:



Now we can introduce the concept of transreal logical space. The transreal logical space can be defined as a trans-vector space (see [1]) in which we can represent a possible world as a sequence of transreal numbers [5]. In other words, a possible world W_a is represented by a sequence

$$\langle \alpha_{a1}, \alpha_{a2}, \alpha_{a3}, \dots \rangle,$$

in which each α_{ak} is a transreal number, interpreted semantically according to what was expressed above, that tells us the Truth value of the proposition α_k (an atomic proposition) at the possible world a. By such considerations, we can say that the atomic propositions are seen as the axes of the transreal logical space.

Then we can say that a transreal logical space $(\mathbb{R}^T)^{\mathbb{N}}$ is the set of all possible worlds, a *continuous* set, seen as points or trans-vectors in the logical space $(\mathbb{R}^T)^{\mathbb{N}}$.

If we are considering possible worlds as points or trans-vectors, then we are able to use geometrical operations on them in such way that these operations can be interpreted logically. An example of this is the concept of a *general transformation in* $(\mathbb{R}^T)^{\mathbb{N}}$, by which we can go from a possible world W_m to a possible world W_n, by means of a transformation T:

$$T(W_m) = W_n \leftrightarrow T\langle \alpha_{m1}, \alpha_{m2}, \alpha_{m3}, \dots \rangle = \langle \alpha_{n1}, \alpha_{n2}, \alpha_{n3}, \dots \rangle.$$

In the expression above, we are saying that W_m goes to W_n by means of the transformation T.

Now we can define a special kind of transformation that is the translation of the concept of *logical inference* into the Transreal Space $(\mathbb{R}^T)^{\mathbb{N}}$. As is well known, a *logical inference* – a first-class object – is an inferential schema that goes from premises to conclusion in such way that *we can never go from True to False, but the opposite is allowed.* Within logical space, in a very general mathematical way of expressing it, we can say that a *logical inference* can be translated as *logical transformation*, such that a *logical transformation* L is a transformation that goes from the possible world W_i to the possible W_j in such way that every component of W_j is greater than or equal with its correspondent in W_i. In other words:

$$L(W_m) = W_n \leftrightarrow L\langle \alpha_{m1}, \alpha_{m2}, \alpha_{m3}, \dots \rangle = \langle \alpha_{n1}, \alpha_{n2}, \alpha_{n3}, \dots \rangle$$

such that:

1) $\alpha_{n1} \geq \alpha_{m1}$;
2) $\alpha_{n2} \geq \alpha_{m2}$;
3) $\alpha_{n3} \geq \alpha_{m3}$;

$\vdots$

The clauses 1), 2), 3), ... guarantee that the truth values of the components of the possible world W_n – a possible world that can be seen as the "conclusion-point" of a logical displacement that begins in W_m – *are truer than the components of the possible world* W_m, and that fact, in some sense, translates into logical space the intuitive idea behind the concept of *logical inference*.

Now I can say some words about a thought experiment and its relation with Transreal logical space and logical transformations.

4 Thought Experiment and Logical Transformation

In this section, I will try to develop my approach to thought experiments. I will follow the ideas presented in the first and second sections of this paper. From such a perspective, a thought experiment will be defined as a logical transformation between an ideal world W_i – a theoretical world in which we have a mathematical structure or "pure concept" – and a concrete world W_r – a world in which we have real or *idealized* measurements and within it we find actual or *idealized* observers. Then, a thought experiment is a postulated displacement of worlds, between which we want to show that *there is or there is not a formal causality.* That is, by means of a thought experiment, we want to demonstrate that an Ideal world acts or does not act on a specific concrete world, the real world or some variant of it. And what I assume here is that formal causality between worlds can be translated into logical space as the concept of logical transformation. Now we can introduce two possibilities:

A) If you want to prove that there is a formal causality between worlds, then we have a *weak thought experiment*;

B) If you want to prove that there is not a formal causality between worlds, then we have a *strong thought experiment.*

In this way, let us suppose that we have a theory Γ given as a set of a denumerable propositions ε_k, namely:

$$\Gamma : \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m, \ldots\}.$$

So, the sequence of transreal numbers

$$\langle \varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{im}, \ldots \rangle$$

is a possible world W_i in which we evaluate the truehood or falsehood of the theory Γ and its components. If there is a formal causality between W_i and W_r, then there is a logical transformation $L(W_i) = W_r$; and we say that such a transformation between W_i and W_r is a *weak thought experiment between* W_i and W_r. In other words:

$$L(W_i) = W_r \leftrightarrow E_w(W_i) = W_r.$$

Let us suppose we prove that the theory Γ is absolutely false in the world W_i. For the sake of simplicity, we represent this fact by the sequence:

$$W_i^\Gamma = \langle -\infty, -\infty, \ldots, -\infty, \ldots \rangle.$$

In this way, a weak thought experiment with Γ is a *logical transformation* between W_i^Γ and a possible world W_r^Γ that has the relation of formal causality with W_r; then, we have:

$$E_w = \left(W_i^\Gamma\right) = W_r^\Gamma \leftrightarrow E\langle -\infty, -\infty, \ldots, -\infty, \ldots\rangle = \langle r_1, r_2, \ldots, r_m, \ldots\rangle,$$

such that:

$$\begin{array}{ll} 1) & r_1 \geq -\infty; \\ 2) & r_2 \geq -\infty; \\ 3) & r_3 \geq -\infty; \\ & \quad \vdots \end{array}$$

Obviously, every absolutely false theory acts formally on every world, except in those worlds where the components of the theory have truth values equal with nullity.

Therefore, the essence of a thought experiment is to be an argument in which premises and conclusion are seen as possible worlds and, for that reason, we can substitute the usual concept of logical inference by the geometric idea of logical transformation.

5 EPR Paradox: A Thought Experiment in Quantum Mechanics

In 1935, Albert Einstein, Boris Podolski and Nathan Rosen published an article whose aim was to demonstrate the incompleteness of Quantum Mechanics [3]. By the incomplete character of Quantum Mechanics, we mean the fact that the mathematical formalism of Quantum Mechanics, with its predictions of measure, does not determine real physical entities: when establishing in theory that the value of a quantity must be a certain real number, this does not imply the actual existence of this quantity; there is no correspondence between the theoretical entities and their predicted values with the observable reality.

In order to demonstrate such incompleteness in Quantum Mechanics, the aforementioned authors used a mental experiment known as the "Einstein, Podolski and Rosen Paradox"- henceforth, abbreviated to EPR Paradox.

Basically, to carry out the EPR paradox, the path is as follows: In Quantum Mechanics, all information that can be obtained from the state of a system is given by the wave function Ψ. More precisely, let a physical quantity A be given, to which we associate the Hermitian operator $\widehat{A}$, which operator, when acting on the *eigenfunction* Ψ_a, determines the following identity:

$$\widehat{A}\Psi_a = a\Psi_a,$$

where a is a real number (an eigenvalue of operator $\widehat{A}$) which will be the measured value of quantity A when it is in state Ψ_a. Therefore, if we postulate that Quantum Mechanics is complete, then the magnitude A will have real existence in state Ψ_a, since its value is predicted by theory for this state.

Let us now consider a physically observable magnitude B, to which we associate the Hermitian operator $\widehat{B}$. In an analogous way to what happens with the magnitude A, when B is in state Ψ_b, then we have the following identity:

$$\widehat{B}\Psi_b = b\Psi_b.$$

In which b is the predicted value of the magnitude B, since this is in state Ψ_b.

Let us postulate, too, that $\widehat{A}$ and $\widehat{B}$ do not commute, namely:

$$\widehat{A}\widehat{B} \neq \widehat{B}\widehat{A}.$$

In this case, we know, according to the Heisenberg Uncertainty Principle, that if we precisely measure the magnitude A, then the magnitude B will have its measurement completely indeterminate at the instant at which the measurement of A is done. Thus, if the theory predicts a precise value of A, for a determinate state, then, for the same state, the theory must predict the complete indeterminacy of B. In this way, the magnitudes A and B cannot have simultaneous physical realities, since we admit the completeness of Quantum Mechanics.

We admit now that two physical systems I and II have interacted in the past through a time interval $T = t_2 - t_1$. In this way, for all instants $t > t_2$, according to the Schrödinger Equation, the state of the "entangled" system $I + II$ can be described in the following way:

$$\Psi_{I+II}\left(x_1, x_2\right) = \sum_{n=1}^{\infty} \psi_n\left(x_2\right) u_n\left(x_1\right),$$

in which x_1 and x_2 are variables associated with the systems I and II, and $u_n(x_1)$ are *eigenfunctions* related to the operator $\widehat{A}$, linked to the system I, with *eigenvalues* a_n; $\psi_n\left(x_2\right)$ are the coefficients of expansion of $\Psi_{I+II}\left(x_1, x_2\right)$ in the orthogonal basis formed by $u_n(x_1)$.

If we measure the magnitude A in the system I, we can find, as the value of the measurement, the *eigenvalue* a_k and, in this case, the state of the system $I + II$ will be the reduced state

$$\psi_k\left(x_2\right) u_k\left(x_1\right).$$

Therefore, after the measurement of A in I, the systems I and II are respectively at the states $u_k\left(x_1\right)$ and $\psi_k\left(x_2\right)$.

Now we can choose, as the orthogonal basis of equation $\Psi_{I+II}\left(x_1, x_2\right)$, the *eigenfunctions* $v_s\left(x_1\right)$, that are connected to the operator $\widehat{B}$ with *eigenvalues* b_s. As said before, we are assuming that $\widehat{B}$ does not commute with $\widehat{A}$. Then, we can represent $\Psi_{I+II}\left(x_1, x_2\right)$ in the following way:

$$\Psi_{I+II}\left(x_1, x_2\right) = \sum_{s=1}^{\infty} \phi_s\left(x_2\right) v_s\left(x_1\right),$$

in which $\phi_s\left(x_2\right)$ are the coefficients of expansion of the *eigenfunctions* $v_s\left(x_1\right)$. If we measure the magnitude B in the system I, we can find, as the measured value, the *eigenvalue* b_r. Thus, the system $I + II$ will be reduced to the system $\phi_s(x_2)\, v_s(x_1)$: the system I is at state $\phi_s(x_2)$, and the system II is at state $v_s(x_1)$.

As we are postulating that $\widehat{A}$ and $\widehat{B}$ do not commute, when A is perfectly determined in the system, then, in the same system, B is completely indeterminate; and when B is perfectly determined in the system, then A is entirely indeterminate in the same system. Then, by the completeness criterion, A and B cannot have simultaneous physical reality according to the Heisenberg Uncertainty principle.

But the *Gedankenexperiment*, shown above, is a thought experiment that uses the concept of an "entangled" system to reveal the following paradoxical situation: what happens at the system I has a direct influence on the possible measurements that can be performed at system II. From the theoretical perspective of the "entangled" Schrödinger Equation, regarding the magnitude A, the probability of system II being in state $\psi_k(x_2)$ is unity, *when a measurement of A is done at I, and the state $u_k(x_1)$ is found as the result of the measurement.* Then, at the same time, *instantaneously*, any measurement of B that could be done at II would be completely indeterminate, *even if no measurement of A at system II is done.* But if no measurements of A is done at II, we can expect that the measurement of B at II would be determinate: if we perform a measurement of B at II, *without measuring* A at II, we will find some definite real number as the result. Thus, without a measurement of A done in II, A and B coexist at II as *potentialities* that express mathematically as probabilities. However, if a measurement of A is done at I, B cannot exist as a potentiality at II, since its measurement is completely indeterminate. But such a paradoxical situation results from the fact that we are admitting the completeness of Quantum Mechanics: the mathematical apparatus of Quantum Mechanics corresponds to determined physical realities. Since from that thesis we can infer a paradoxical situation, we must deny the completeness of Quantum Mechanics, and that was the conclusion from that *Gedankenexperiment* performed by Einstein, Podolsky and Rosen – the famous EPR paradox.

In fact, we can present the EPR paradox in a more analytical way. From this perspective, the paradox emerges from the fact that we are admitting the completeness of Quantum Mechanics, along with the fact that such completeness implies the simultaneous existence in reality of predicted theoretical entities, would result in the possibility of a simultaneous measurement of these theoretical entities that are predicted by the theory. But the Heisenberg Principle denies such simultaneous measuring; then, we must deny the completeness of Quantum Mechanics.

The Brazilian philosopher Silvio Chibeni presents an explicit derivation of the EPR Paradox, built within propositional calculus. Chibeni's derivation is as follows, using the predicates

C: the quantum mechanical description of reality is complete;
SR: conjugate quantities can have simultaneous reality;
QM_{AB}: Quantum Mechanics affords simultaneous precise values to the conjugate quantities A and B.

1.	$(SR \,\&\, C) \to QM_{AB}$	[completeness criterion]
2.	$\neg(QM_{AB})$	[QM: Heisenberg Uncertainty Principle]
3.	$\neg(SR \,\&\, C)$	[1 and 2]
4.	$\neg C \vee \neg SR$	[3]
5.	$C \to SR$	[reality criterion applied to the correlated pairs]
6.	$C \to \neg SR$	[4]
7.	$C \to (SR \,\&\, \neg SR)$	[5 and 6]
8.	$\neg C$	

([2], pp. 5–6).

Then, we can infer that Quantum Mechanics is not complete, since this assumption lead us to a paradox: conjugate quantities – quantities that don't commute – can have simultaneously reality and cannot have simultaneously reality.

6 A Transreal Analysis of EPR Paradox

Let me now present a version of EPR Paradox embedded into the concept of logical transformation. First of all, I will consider the EPR Paradox as the conclusion that *there is no logical transformation between an ideal version of Quantum Mechanics and its concrete version in which there is an "entangled" system.* In other words, the EPR Paradox, in its Transreal version, shows that *there is no formal causality between an ideal model of Quantum Mechanics and its "entangled" model.* Therefore, EPR is a *strong thought experiment.*

Let us start the analysis of the EPR paradox by assuming that the ideal model of Quantum Mechanics is a world in which no measurement is done, and all we have is the mathematical apparatus of Quantum Mechanics. We also admit that we can express the ideal model of Quantum Mechanics as an enumerable list of proposition in which we can find what we must expect about the states concerning two conjugate quantities A and B at two system I and II *that do not interact each other.* Thus, our ideal model of Quantum mechanics will be:

$$Q^I = \langle q_{i1}, q_{i2}, \ldots, q_{im}, \ldots \rangle.$$

Regarding the system II, there is a q_{km}, a transreal number that belongs to $\mathbb{R}^T \setminus \{\Phi\}$, that can be associated with the proposition:

$$v(\Psi\langle \widehat{B}; II; a\rangle) = q_{km}.$$

The proposition above says that the truth value of the proposition that says that the operator $\widehat{B}$; regarding the system II, if *some measurement is done*, has the *eigenvalue a* is equal with q_{km}. This number could be any transreal number, except nullity: in this case, the mentioned physical description of the state of the system II regarding the magnitude B will be completely indeterminate, but this case would occur if, and only if, a measurement of the conjugate quantity A was done at II at same time when the measurement of B is being performed.

Now let's consider a concrete world or model Q^C in which we have the "entanglement" of the system I and II and we have also an idealized observer O that performs a measurement of A at the system I. As shown above in the EPR paradox, if the measurement of A gives to us the state

$$\Psi\langle\widehat{A}; I; k\rangle,$$

then, by some kind of *superluminal causality*, instantaneously, the magnitude A at system II has the determinate *eigenvalue* k'. Therefore, the following conditional is absolutely true in the model Q^C.

$$\text{If } v\left(\Psi^{\circ}\langle\widehat{A}; I; k\rangle\right) = \infty, \text{ then } v\left(\Psi\langle\widehat{A}; II; k\rangle\right) = \infty.$$

Thus, since the ideal observer O does the measurement of A at I and finds the *eigenvalue* k, then we can affirm that

$$v\left(\Psi\langle\widehat{A}; II; k\rangle\right) = \infty.$$

But, in this case, we have necessarily by Heisenberg's Uncertainty Principle that:

$$v\left(\Psi\langle\widehat{B}; II; a\rangle\right) = \Phi.$$

Since A and B are conjugate quantities whose operators do not commute, if A has a determinate *eigenvalue* at II, simultaneously the correspondent *eigenvalue* of B is completely indeterminate.

Now let us postulate that there is a logical transformation G between the worlds Q^I and Q^C, in such way that equal propositions occupy the same position in the sequences corresponding to Q^I and Q^C respectively. Then, if there is logical transformation $G(Q^I, Q^C)$, we expect that:

$$G\left(Q^I\right) = Q^C \leftrightarrow G\langle q_1, q_2, \ldots, q_i, \ldots\rangle = \langle g_1, g_2, \ldots, g_i, \ldots\rangle$$

in such way that:

1) $g_1 \geq q_1$;
2) $g_2 \geq q_2$;
3) $g_3 \geq q_3$;

$\vdots$

Clearly, such a logical transformation does not exist. To prove this, let us consider that the proposition $\Psi\langle\widehat{B}; II; a\rangle$ occupies the m-th position in the possible world Q^I and must occupy the same position in the possible world Q^C. But

$$v(\Psi\langle\widehat{B}; II; a\rangle) = \Phi \geq v(\Psi\langle\widehat{B}; II; a\rangle) = q_m,$$

in which

$$q_m \in \mathbb{R}^T \setminus \{\Phi\},$$

is obviously false in transreal arithmetic, and the condition for the existence of such transformation between Q^I and Q^C, as presented above, is not satisfied.

Then, in its Transreal version, EPR paradox shows that there is no formal causality between Q^I and Q^C; then $G(Q^I, Q^C)$ is a *strong thought experiment between* Q^I and Q^C.

7 Conclusion

In this small article, I present an analysis of the EPR paradox, a thought experiment, from the concept of logical transformations in transreal logical space. Such a general analysis could be an epistemic instrument to be used in Philosophy of Science and in the methodology of science. With the aid of this very general concept of thought experiment, based on transreal numbers, the clarifying of what a thought experiment is might be achieved.

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