Transmathematica 2017 - Transmathematics and The Philosophy of Numbers

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Abstract

We present the call for papers and abstracts of the first Transmathematica conference, which was held in 2017 as Session 83, Transmathematics and the Philosophy of Numbers, of the 25th International Congress of History of Science and Technology (ICHST), Rio de Janeiro, Brazil, 23-29 July. In the absence of the present paper, the abstracts would otherwise have been lost from the public record.

1 Introduction

The first Transmathematica conference was held in 2017 as Session 83 of the 25th International Congress of History of Science and Technology (ICHST), Rio de Janeiro, Brazil, 23-29 July. The Session was called "Transmathematics and the Philosophy of Numbers." We have added "Transmathematica 2017" to the title of the present paper to make it clear that this conference was the first of a series of conferences.

Prior to the conference, a publisher agreed to host the Transmathematica journal, with a view to launching the journal with the conference papers. However, the publisher withdrew, at short notice, saying they had received advice from the scientific community that Transmathematics should not be supported. The conference went ahead but the only record of papers was the ICHST abstracts. These abstracts were linked to from various websites.

Following the conference, the Transmathematica journal was established using the Open Journal System (OJS) and published its first issue to coincide with Transmathematica 2019 - The 2nd International Conference on Total Systems. However, by this time, the web links to the abstracts of the 2017 conference were broken so it seemed the abstracts had been lost. However, James Anderson had preserved a personal copy of the official abstracts so we are able to present both the call for papers and the abstracts here.

The call for papers is in the next section, Section 2. The abstracts run from Section 3 to Section 7. All of these sections have been reformatted from the original and minor typographical errors have been corrected. The result is a faithful, but not exact, record of the original.

2 Call for Papers

Session 83 Transmathematics and the Philosophy of Numbers (2 sessions, 10 participants)

The History of Mathematics

Organizers:

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Abstract:

Numbers have long played a critical role in everyday life, commerce and the sciences. The recent development of transmathematics extends number systems so that they have no exceptional states.

This symposium provides a forum for discussion between professionals from different areas of knowledge, such as mathematics, logic, philosophy, history of science, computer science, physics, and any other specialism that is concerned with numbers, their contemporary or historical applications, or their geographical spread through societies. The development of new number systems is a recurring event in the history of mathematics and science. Some categories of numbers, for example, negative, irrational, imaginary and infinitesimal numbers, were initially introduced as transient entities, which appeared during calculations but were not numbers themselves; their appearance was always conditioned to the "actual" numbers. But as these new objects became increasingly common, disparaging them was no longer tenable. Then people sought ways to interpret them, trying to fit them to the categories of numbers that were already accepted. Thus geometrical, algebraic, topological, computational and many other interpretations of arithmetic were developed, whose operations were similar to existing arithmetics. At several times in history, mathematics went through epistemological discussions legitimising its ideas. The very concept of number is steeped in these discussions. Now transreal numbers are being proposed and developed as an advance on our computational abilities and on our mathematical and scientific understanding. It is a truism to say that the universe never stops; that all manner of physical interactions take place without the universe ever stopping to consult an external oracle to decide what should happen next. But this is precisely what does happen in computers that monitor or control aspects of the universe. Such computers generally fail on some kind of exception and, after exhausting any special programming, they halt in an error state, until they are reset by an external agency, typically a human being. Computer errors are such a normal part of our lives that we seldom doubt that it is in the nature of things that such errors exist. But we should doubt. If, as we suppose, the universe operates without exceptions, why should a small part of it, a computer, have the privilege of exceptional behaviour? How can computers have the power to fail? Can we arrange that they do not fail or that they fail less often? Does transmathematics provides this ability? Does it provide a mathematical route to understanding the physics of the universe, even at singularities?

Extemporaneous laws, like the prohibition of division by zero, always charge a price that appears as impossibilities, whose solution is the creation of what should always have been. Is transreal nullity, the unique number zero divided by zero, like the presence of the undecidable; necessary for the good functioning of the real numbers? Did Gödel predict the invention of the nullity, not in his famous theorems, but in the philosophical inventions arising from its results?

We invite papers on any aspect of transmathematics or the philosophy of numbers and look forward to a provocative and productive discussion at the symposium.

Keywords: Transmathematics, Philosophy of Numbers, History of Mathematics and Science, Transreal Numbers, Computer Science.

Expected Participants:

1. Ricardo Kubrusly, Brazil, Federal University of Rio de Janeiro

2. James Anderson, United kingdom, University of Reading

3. Tiago Reis, Brazil, Federal Institute of Education, Science and Technology of Rio de Janeiro

4. Walter Gomide, Brazil, Federal University of Mato Grosso

5. Vassil Alexandrov, Spain, Barcelona Supercomputing Center

6. Paul Johnson, Hong Kong, Kontel Microsystems

7. Carlos Roberto, Brazil, Federal Institute of Education, Science and Technology of Rio de Janeiro

8. Renata Barros, Brazil, Federal Institute of Education

9. André Senra, Brazil, Science and Technology of Rio de Janeiro

10. Magno Ferreira, Brazil, Federal Institute of Education, Science and Technology of Rio de Janeiro

3 Semiotics of Mathematics

"Semiotics of Mathematics: Understanding how symbolism can change our perception of an object" by Dorival Rordrigues da Rocha Junior, Universidade Federal do Rio de Janeiro. Key words: Semiotic, Symbolism, Transmathematics.

Mathematical language is a powerful set of symbols ruled by logic. It is used as a credibility certificate in the sciences and extends to common sense. Doing mathematics is a mental process, yet mathematicians manipulate a semiotic system of symbols to do their work. The mental activity deals with abstract entities, manipulated according to a logical structure. When these entities are described on paper, one might wonder how veridical the description is to the abstract object or how the script might affect that object.

Symbolism is normally seen as an abstract doing; however, Tall (A Versatile Theory of Visualisation and Symbolisation in Mathematics, 1994) says as symbols are written and seen, they are more than mental reflection. Marks on paper allow deeper thoughts on symbols, bringing up others perceptions over the object. Manipulation of symbols is connected to the way one does mathematics. Some processes, expressed in symbols, can turn into objects, to be manipulated; what Tall calls procept, when expressions represent either a process or a concept.

Rotman (Toward a Semiotics of Mathematics, 2000) tries to reconcile written symbols and abtract ideas. He states that a finite, human Subject takes the marks on paper, the mathematical symbols, transports them to an imaginary mental world, containing an Agent, where finite and infinite mathematics are possible, and then a mental projection of the Subject into the Agent process this information. Some manipulations are finite and can be done by the Subject alone but only the Agent can access the idea of tending to a limit, for example; although the Subject may divide one by a number, only the Agent can truly perceive the infinitary activity the concept of limit holds. Given Rotmans structure, an individual reading mathematical text creates this Agent to perform the actions and then the Subject registers it. According to Rotman what validates mathematical work is the Person, an entity with time and cultural background that combines the Subject and Agent. Transmathematics claims that signs are potential activity of a mathematical Subject. Its semiotic representation gives arise to new kinds of perceptions for old concepts, such as $x + 1$, or infinity, what was once seen as a process gets objectified. One might have wondered what 0 over 0 means, that can be a completely different thought when a Person visualizes a point floating over a line. These representations change perception on known objects.

4 Naive Set-Theory Without Paradox!

"Naive Set-Theory Without Paradox!" by James Anderson, University of Reading. Key words: Transreal numbers, Russell's Paradox, set theory.

Sets and numbers influenced each other in the history of mathematics. We now show how set theory can respond to the transreal numbers.

We adopt first-order logic, with equality, $=$, as a base language. In modern terms naive set-theory takes $\{x \mid \phi(x)\}$ to be a set, defined by the class $\phi(x)$. The Russell Paradox shows this is inconsistent. Historically this inconsistency was barred by type theories or syntactic restrictions on sets. No such barring is necessary!

We adopt naive set-theory, as a class theory, with a Universal Class, U, partitioned into the Universal Set, V, and the Universal Antinomy, W, such that $x \in V \iff x = x$ and $x \in W \iff x \neq x$. We assert an equivalence operator, interchangeability, $\equiv$, whose base case is that two objects, in our class theory, are interchangeable if their defining sentences, in the base language, are identical. We define a special set, infinity, $\infty = V \setminus \{V\}$ and a special antinomy, nullity, $\Phi \equiv W \setminus \{W\}$. The usual constructions of the natural and ordinal numbers now construct transnatural and transordinal numbers and none of the usual paradoxes of set theory hold. As an example we dissolve the Russell Paradox. $R_w \equiv \{x_1 \mid x_2 \notin x3\}$ – is the Russell Antinomy, when $x_1 \equiv x_2 \equiv x_3$. Suppose $x_1 \equiv R_w$, then $R_w \in R_w$, whence $R_w \notin R_w$, by $x_2 \notin x_3$. Thus $R_w \in R_w \implies R_w \notin R_w$. Conversely suppose $x_2 \equiv x_3 \equiv R_w$, then $R_w \notin R_w$, whence $R_w \in R_w$, by x_1. Thus $R_w \notin R_w \implies R_w \in R_w$. Combining implications we have the classical paradox: $R_w \in R_w \iff R_w \notin R_w$.

The Axiom of Extensionality states: $x = y \implies (z \in x \implies z \in y)$. Taking $x = y = z = R_w$, gives $R_w = R_w \implies (R_w \in R_w \implies R_w \in R_w)$ but we have $R_w \in R_w \iff R_w \notin R_w$. Therefore $R_w \neq R_w$, so R_w is an antinomy. How do we dissolve Russells Paradox? We cannot assert R_w does not exist, because it exists as an antinomy. We cannot assert one of $R_w \in R_w$ or else $R_w \notin R_w$ because, in either case, the paradox would be a contradiction. The Axiom of the Excluded Middle blocks the dialetheia: $R_w \in R_w$ & $R_w \notin R_w$. What remains? A gap remains: $R_w \in R_w$ has no degree of truth or falsehood. We define that $R_v = R_w \cap V$ is the Russell Set. Some easy, non-paradoxical theorems follow, including: $R_w \notin R_v; R_v \notin R_v; R_v \in R_w$!

5 Dynamics of Science Construction

"A Look at the Dynamics of Science Construction Through Transreal Numbers" by Isabel Cafezeiro, Universidade Federal Fluminense and André Campos da Rocha, HCTE - UFRJ and Carmem Gadelha, HCTE - UFRJ and Ricardo Kubrusly, HCTE - UFRJ. Key words: Mathematics, state science, nomadic science.

We realize that certain subjects (or approaches) of science occupy a place of clear disadvantage as if they were a demerited science. At the same time, another set of themes receives special treatment. This configures a permanent tension and, nevertheless, an interdependence between that "demerit" science and the other, a "noble" science. Everything happens as a rivalry, but with moments of alliance and agreement. This triggers modes of operation and division of labor and resources in the academic world, which affects us daily.

Throughout this text we deal with the transreal numbers, both in the metaphoric sense as in the naming given to the numerical set. In the first case, we assign all scientific ambition to the perfection and radiance of something that goes beyond the real; In the second, it is really the numerical set itself. Analyzing the course of conception and definition of the transreal numbers we call attention to processes that, in dealing with the wandering in the world, refuses any framework within the limits of a mathematics "queen of the sciences."

6 Transreal Numbers and Possible Worlds

"Transreal Numbers and Possible Worlds" by Tiago Soares dos Reis, Federal Institute of Education, Science and Technology of Rio de Janeiro. Keywords: transreal numbers, total semantics, possible worlds, hypercyclic operators.

Elsewhere I, with my co-workers J. Anderson and W. Gomide, use total semantics to obtain a geometrical Possible World Space that models all possible worlds. Total semantics is a logical system that uses transreal numbers to set a semantics that contains classical, paraconsistent, fuzzy and indeterminate values. Our Possible World Space is a Cartesian co-ordinate frame, where each axis is an atomic proposition and every point is a possible world whose co-ordinates are the semantic values of its propositions. Using hypercyclic operator theory we prove the existence of worlds which approximate every world by repeated application of a single operator. That is we prove the existence of universal, possible worlds.

Now I have two research students. We are researching the Possible World Space. Some results are already known from hypercyclic operator theory: (1) if X is a space which has an hypercyclic operator then every vector from X is the sum of two hypercyclic vectors; (2) if T is an hypercyclic operator then any linear combination between iterates of T has a dense image; (3) if T is an hypercyclic operator and x is an hypercyclic vector then every element from the subspace generated by the orbit of x is an hypercyclic vector; and (4) the set of hypercyclic vectors generated by T is equal to the set of hypercyclic vectors generated by T_n for all positive integers n.

We are investigating whether our Possible World Space has the above properties. If so, it follows that, respectively: (1) every possible world is the sum of two universal worlds; (2) every possible world can be approximated by the image of linear combinations between iterates of an hypercyclic operator; (3) the subspace generated by the orbit of a universal world is made of universal worlds; and (4) the set of universal worlds generated by T is equal to the set of universal worlds generated by T_n for all positive integers n.

The second property allows continuous proof paths over hypercyclic iterates. It allows searches and optimisation. The fourth property means that if we generate a counterfactual world from a universal world then we can still get arbitrarily close to any world, in other words actions can be reversed. This gives a mathematical justification for a person to engage in good actions and it justifies atonement for sin.

7 Naive Transset Theory

"Naive Transset Theory and a Proposal of an Alternative Methodology of Science" by Walter Gomide, Federal University of Mato Grosso - UFMT. Key words: Transreal, Transset, Logic.

Naive transset theory, created and being developed by James Anderson, deals with two disjoint classes, V (sets) and W (antimonies). Each one of these two branches has its own logic, in which the Axiom of Excluded Middle holds: the universe of sets V is classical, and the diagonal $-\infty, \infty$ of Transreal Square of Opposition is adequate to be its semantic background in such way that the Law of Excluded Middle does not allow any other truth value than True or False; the universe of antinomies W is non-classical, and the diagonal $0, \infty$ of Transreal Square of Opposition is adequate to be its semantic background in such way that the Law of Excluded Middle does not allow any other truth value than "dialetheia" or "gap." So we can avoid Russell Paradox, in the extent that this statement deals with an antinomy and, for this reason, the Axiom of Excluded Middle implies that Russel Paradox should be really paradoxical or a gap. If you discharge the possibility of being a real paradox (what is reasonable - paradox implies inconsistency), then gap value remains, and the paradox is solved.

Further we can see that all sets and all antinomies have their "conceptual" counterparts. We can see that these counterparts are properties that divided the universe of concepts or properties into two disjoints sets: the set V^* of properties that obey the Law of Non-Contradiction, and the set W^* of properties or concepts that don't obey Law of Non-Contradiction, and these two sets have the same cardinality as proved by Anderson. Then we can admit that for every classical concept (these ones that obey Law of Non-Contradiction) we can correlate a non-classical concept (these ones that don't obey the Law of Non-Contradiction), which one is the image or a representation of a classical concept.

Theses images or representations of classical concepts or properties could be seen as the imaginary source of objective concepts, and the logic adequate to deal with them is non-classical, once they correspond to the non-classical diagonal of Transreal Square of Opposition.

The idea here is present a methodology of science based upon this distinction between classical and non-classical properties that arises from Naive transset theory.

8 Conclusion

We present the call for papers and abstracts of the first Transmathematica conference, which was held in 2017 as Session 83, Transmathematics and the Philosophy of Numbers, of the 25th International Congress of History of Science and Technology (ICHST), Rio de Janeiro, Brazil, 23-29 July. In the absence of the present paper, the abstracts would otherwise have been lost from the public record.

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